Key (08/09)

Activity 1: Properties of Exponents

Recall that when solving any equation, you have basically three methods which you can use: graphs, tables, and symbolically (i.e. by hand).

You can solve the following two equations symbolically:

$$2x - 8 = 5(1 - x) + 2 \qquad\qquad 4x^2 - 10 = 190$$

1. Which of the two equations above is ready to be solve immediately "by hand?" Explain why.

   $4x^2 - 10 = 190$ Order of ops -- reverse & undo.

2. What must be done to the other equation above before you can solve "by hand?"

   Need to simplify first.

Being able to simplify is very important. Not only does it allow you to solve equations, but it offers you a shorter, less complicated looking expression.

3. Simplify each of the following expressions so that the result is equivalent and short as possible.

post

a. $3x^4 + 9x^2 - 9 - 8x^2 + 3x$

   $3x^4 + x^2 + 3x - 9$

b. $5x - 9 - (7 - 3x) + 10x$ $\qquad 18x - 16$

   $5x - 9 - 7 + 3x + 10x$

c. $x(9 - 8x) + 3x + 7$ $\qquad -8x^2 + 12x + 7$

   $9x - 8x^2 + 3x + 7$

d. $7am + 9am^2 - 9m + 3ma + 6m$ $\qquad 9am^2 + 10am - 3m$

Notice that the main operation in each expression above is addition (and subtraction which is just "adding the opposite"). But how would these expressions and their results change if the main operation between terms was multiplication? You will explore some examples and see what happens.

Before you begin to study some patterns to shorten expression involving multiplication as the main operation, it is important to remember the following:

Multiplication is the shorthand way to write repeated addition of the same number.

ex. $5 + 5 + 5 + 5 + 5 + 5 + 5 = 7 \bullet 5$

Exponents are the shorthand way to write repeated multiplication of the same number.

ex. $5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 = 5^7$

Activity 1: Properties of Exponents

4. Use that information to shorten each of the given expressions without actually determining what they are equal to. Pay attention to the operation …

a. $4+4+4+4+4$ $4 \cdot 5$

b. $\left(\frac{7}{3}\right)\left(\frac{7}{3}\right)\left(\frac{7}{3}\right)\left(\frac{7}{3}\right)$ $\left(\frac{7}{3}\right)^4$

c. $m \bullet m \bullet m \bullet x \bullet x$ m^3x^2

d. $m \bullet m \bullet m + x \bullet x$ m^3+x^2

e. $-9 \bullet -9$ $(-9)^2$

f. $-9 + -9$ $2(-9)$

Experiment 1: The Product of Powers

5. Consider this expression: $5^2 \bullet 5^4$
   a. As written, there appears to be two calculations which are being used. What two calculations are they?
   multiplication & exponents
   b. If you really think about it, what is the one and only operation which is really being used?
   multiplication
   c. Explain your response to part b.
   exponents are the short version of repeated multiplication

6. Based upon what you know about multiplication and exponents, the expression in #5 can be written in the following "long' from:

$$5^2 \bullet 5^4 = (5 \bullet 5) \bullet (5 \bullet 5 \bullet 5 \bullet 5)$$

Note that the parenthesis is not needed. They are put there is emphasize the two different exponents which are being used!!

Based upon how the original expression has been written in "long" form, what would the expression, $5^2 \bullet 5^4$, be equivalent to in "short" form?

$5^2 \bullet 5^4 = 5^6$

Activity 1: Properties of Exponents

7. Complete this table showing the "long" and "short" forms of various expressions. Include the parenthesis in your "long" forms!!

Original Expression	"long" form	"short" form
$5^2 \bullet 5^4$	(5•5)•(5•5•5•5)	5^6
$2 \bullet 2^6$	$(2)\cdot(2\cdot2\cdot2\cdot2\cdot2\cdot2)$	2^7
$B^4 \bullet B^8$	$(B\cdot B\cdot B\cdot B)(B\cdot B\cdot B\cdot B\cdot B\cdot B\cdot B\cdot B)$	B^{12}
$m^4 \bullet m \bullet 9m^4$	$(m\cdot m\cdot m\cdot m)\circ m\cdot 9(m\cdot m\cdot m\cdot m)$	$9\cdot m^9$
$x^2 \bullet y^5$	$(x\cdot x)\circ(y\cdot y\cdot y\cdot y\cdot y)$	$x^2\cdot y^5$

8. Which original expression didn't become any shorter? Explain why.

last one → bases were not the same (x vs. y)

9. Study all the other "original' expressions and their respective "short" forms. Describe the pattern which you see.

If the base are the same and multiplied, then keep the base and add the exponents to find your new exponent.

10. Based upon the pattern you just described, write this expression in "short" form.

$x^m \bullet x^n = x^{m+n}$

11. Notice that the title of the experiment was "**Product of Powers.**" Explain why.

Powers (like bases) are multiplied --- add exponents

Before you begin the next experiment, do the following:

post

12. Solve the following equation for x: $4x = -17$

$x = -17/4$

13. How did you solve this equation for x? Why did you use this method?

divided by 4 --- it undoes the mult. to x.

14. Simplify each of these expressions ....

a. $8 \div 8 = 1$ b. $0.5 \div 0.5 = 1$ c. $-9 \div -9 = 1$

15. Why did you get the results you did in the previous question?

one for all.

Activity 1: Properties of Exponents

16. Reduce the given fractions be **rewriting** both the numerator and denominator as the product of other values. **Then reduce** the fraction by "dividing out" everything you can.

a. $\frac{6}{8}=\frac{2\bullet 3}{2\bullet 4}=\frac{3}{4}$ b. $\frac{50}{2}=\frac{2\cdot 25}{2}=25$ c. $\frac{27}{81}=\frac{3\cdot 3\cdot 3}{3\cdot 3\cdot 3\cdot 3}=\frac{1}{3}$

Experiment #2: The Quotient of Powers

Post

17. Complete the table using what you know about the meaning/use of exponents and also what you know about reducing fractions. Do NOT simplify the numerical expression to what they equal.

Original Expression	"long" form	"short" form
$\frac{3^4}{3^2}$	$\frac{3\cdot 3\cdot 3\cdot 3}{3\cdot 3}$	3^2
$\frac{10^7}{10^4}$	$\frac{10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10}{10\cdot 10\cdot 10\cdot 10\cdot}$	10^3
$\frac{m}{m^8}$	$\frac{m}{m\cdot m\cdot m\cdot m\cdot m\cdot m\cdot m\cdot m}$	$\frac{1}{m^7}$
$\frac{4b^{10}}{2b^9}$	$\frac{2\cdot 2\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b}{2\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot}$	$2\cdot b$
$\frac{x^5}{y^3}$	$\frac{x\cdot x\cdot x\cdot x\cdot x}{y\cdot y\cdot y}$	$\frac{x^5}{y^3}$

18. Which original expression didn't become any shorter? Explain why!

last one → not like bases so I couldn't cancel anything out.

19. Study all the other "original" expressions and their respective "short" forms. Describe the pattern which you see. If like bases are being divided, keep the base and find the difference between the top exponent & bottom exponent. (top − bottom).

20. Based upon the pattern you just described, write this expression in short form:

$$\frac{y^m}{y^n}= y^{m-n}$$

Activity 1: Properties of Exponents

21. Notice that the title of this experiment was "**Quotient of Powers**." Explain why.

Two powers are being divided.

22. Recall that exponents are the shorthand way to write repeated multiplication. Sometimes these repeated multiplications are more complicated. Consider these two expressions:

$$\mathbf{(3m)^2} \text{ and } \mathbf{3m^2}$$

The parentheses are important. The parentheses are needed to help you recognize what the base is. For example, the expression to the left has a base of 3m while the expression to the right has a base of only m. in other words ....

$$(3m)^2 = (3m) \bullet (3m) \qquad \text{and} \qquad 3m^2 = 3 \bullet m \bullet m$$

Though both expressions have a 3, an "m", and an exponent of "2", the results are very different depending upon the use of the parentheses.

Pay close attention to the bases, write each of these expressions in "long" form only.

a. $(2m)^4$ $= (2m)(2m)(2m)(2m)$

b. $6x^7$ $= 6 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot$

c. $9.8 \bullet 10^3$ $= 9.8 \bullet 10 \cdot 10 \cdot 10$

d. $(a + b)^2$ $= (a+b)(a+b)$

Experiment 3: The Power of a Power

23. Complete this table. Do not simplify numerical expression down to what they equal. Be attentive to the location of the various exponents. The first expression has been partially completed for you.

Original Expression	"long" form	"short" form
$(5^4)^2$	(5•5•5•5) • (5•5•5•5)	5^8
$(2^3)^5$	$(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$	2^{15}
$(m^2)^4$	(mm)(mm)(mm)(mm)	m^8
$(b^4)^3$	(bbbb)(bbbb)(bbbb)	b^{12}

Activity 1: Properties of Exponents

24. Study the original expression and its equivalent, short form. Describe the pattern whish is observed.

When a power is raised to an exponent (power), keep the base and multiply the exponents. → raise the base to the product

25. Based upon your description, what would the given expression simplify to?

$$(a^m)^n = a^{m \cdot n}$$

26. This pattern is called the "**power of a power.**" Explain why.

Inside the parenthesis is called a power and exponents are also sometimes called the power. ∴ a power raised to a power.

Experiment 4: The Power of a Product

27. Complete this table. Do not simplify the numerical expressions all the way down to what they equal. Your final "equivalent" form should be written **differently** than the original expression.

Original Expression	"long" form	"short" form
$(6 \bullet 5)^3$	(6•5) (6•5) (6•5)	$6^3 \cdot 5^3$
$(2x)^5$	$(2x)(2x)(2x)(2x)(2x)$	$2^5 x^5$
$(\pi d)^2$	$(\pi d)(\pi d)$	$\pi^2 d^2$
$(ab)^4$	$(ab)(ab)(ab)(ab)$	$a^4 b^4$

28. What is the only operation which is going on inside of the parentheses?

multiplication

29. Study the original expression and its equivalent short form. Describe the pattern which is observed.

each factor inside parenthesis is raised to the exponent the parenthesis is raised to.

30. Based upon your description, what would the given expression simplify to?

$$(xy)^m = x^m y^m$$

31. This pattern is called the "**power of a product.**" Explain why.

The product of 2 factors is raised to the exponent. (the base)

Activity 1: Properties of Exponents

Experiment 5: Power of a Quotient

32. Complete this table. Do not simplify the numerical expressions all the way down to what they equal. The first one has been done for you. **Recall** that when multiplying fractions, you simply multiply all the numerators (i.e. multiply across the "top") and multiply all the denominators (i.e. multiply across the "bottom").

Original Expression	"long" form	"short" form
$\left(\frac{2}{7}\right)^4$	$\left(\frac{2}{7}\right)\bullet\left(\frac{2}{7}\right)\bullet\left(\frac{2}{7}\right)\bullet\left(\frac{2}{7}\right)=\frac{2\bullet 2\bullet 2\bullet 2}{7\bullet 7\bullet 7\bullet 7}$	$\frac{2^4}{7^4}$
$\left(\frac{1}{4}\right)^2$	$\frac{1}{4}\cdot\frac{1}{4}=\frac{1\cdot 1}{4\cdot 4}$	$\frac{1^2}{4^2}$
$\left(\frac{m}{6}\right)^5$	$\left(\frac{m}{6}\right)\left(\frac{m}{6}\right)\left(\frac{m}{6}\right)\left(\frac{m}{6}\right)\left(\frac{m}{6}\right)$	$\frac{m^5}{6^5}$
$\left(\frac{w}{y}\right)^3$	$\left(\frac{w}{y}\right)\left(\frac{w}{y}\right)\left(\frac{w}{y}\right)$	$\frac{w^3}{y^3}$

33. What is the only operation which is going on inside of the parentheses?

division

34. Study the original expression and its equivalent short form. Describe the pattern which is observed.

the quotient inside the parentheses

35. Based upon your description, what would the given expression simplify to?

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$$

36. This pattern is called the "**power of a quotient**." Explain why.

a quotient is what is ~~raised~~ the base.

Activity 1: Properties of Exponents

Experiment 6: Negative Exponents

37. Using the appropriate keys on your calculator to help you, determine the fraction equivalent to each of the following.

Original Expression	fractional expression
4^{-1}	$1/4$
5^{-2}	$\frac{1}{25}$
10^{-3}	$\frac{1}{1000}$
$\left(\frac{2}{3}\right)^{-1}$	$\frac{3}{2}$
$\left(\frac{7}{4}\right)^{-2}$	$\frac{16}{49}$

38. Two things occur when you have a base raised to a negative exponent. Based upon your results above, describe what those two things are.

a. the base changes to it's reciprocal

b. the exponent becomes positive

39. Using the pattern, rewrite this expression so that it no longer contains a negative exponent.

a^{-b}

40. Using the pattern, rewrite this expression so that it no longer contains a negative exponent.

$\left(\frac{w}{y}\right)^{-n} = \left(\frac{y}{w}\right)^{n} = \frac{y^n}{w^n}$

Activity 1: Properties of Exponents

Experiment 7: Zero Exponents

41. Using the appropriate key on your calculator, simplify each expression.

Original Expression	~~fractional~~ simplified expression
10^0	1
$(-2.6)^0$	1
$\left(\frac{2}{17}\right)^0$	1

42. Describe the pattern which you are observing.

any base raised to an exponent of zero = 1

43. Use the pattern to simplify this expression:

$M^0 = 1$

Recall: $(a+b)^2$ from our quadratic work.. In our last 2 experiments, we used multiplication and division inside a set of parentheses with an exponent outside. People are often too quick to say "distribute" as soon as they see the exponent. Be Careful!!

Does $(a+b)^2 = a^2 + b^2$?? Test this theory. pick an $a = \_$ $b = \_$

$(a+b)^2 = (\_ + \_)^2$ $\quad a^2 + b^2 = \_^2 + \_^2$

$= (\quad)^2$ $\quad = \_ + \_$

$= \_\_$ $\quad = \_\_$

Do you get the same answer??

Can you distribute the exponent when there is addition inside the parenthesis??

Activity 1: Properties of Exponents

Summarize the Math

a. What is the definition of an exponent?

repeated multiplication

b. How do you simplify an expression which has a negative exponent? Give an example.

① take the reciprocal of the base
② change the exponent to positive

$x^{-2} = \frac{1}{x^2}$

c. How do you simplify an expression which has a zero exponent? Given an example.

any base raised to the zero = 1

$(x+2)^0 = 1$

d. Consider the following expressions: i. $(2x)^{-2}$ ii. $2x^{-2}$ iii. -16^3

- What is the base of the -2 exponent in the first expression? $2x$
- What is the base of the -2 exponent in the second expression? x
- What is the base of the 3 exponent in the third expression? 16

e. **Simplify** each of the following and **name** the exponent property illustrated in each.

- $(5m^3)^2 \bullet m^4$ $5m^3 \cdot 5m^3 \cdot m^4 = 25m^{10}$

  power of a product ; product of powers ; power of a power

- $8x^0 \bullet (3x)^0$ $8 \cdot 1 \cdot 1 = 8$

  zero exponents

- $\left(\frac{4x^2}{x^3}\right)^{-3}$ $\left(\frac{x^3}{4x^2}\right)^3 = \left(\frac{x}{4}\right)^3 = \frac{x^3}{4^3} = \frac{x^3}{64}$

  negative exponents ; quotient of powers ; power of quotient ;