WARMUP

Advanced Algebra – Sequences & Series Name _____

Date _____

1. The first two terms of a sequence are 7 and 14.

   a. Write the first 5 terms of the sequence if the sequence is arithmetic.

   _____, _____, _____, _____, _____

   b. Write the first 5 terms of the sequence if the sequence is geometric.

   _____, _____, _____, _____, _____

2. A small object that is dropped from the top of a tall building falls 16 ft during the first second, 48 ft during the second, 80 ft during the third second, 112 ft during the fourth second, and so on.

   a. Find the formula that will let you calculate the distance the object falls during the nth second.

   b. Use the formula to determine the distance the object falls during the 10^{th} second.

3. The first row in a semicircular arena contains 72 seats. Moving back, each row has two more seats than the row in front of it. How many seats are there in the 50-row arena?

4. A contractor failed to meet his deadline for completing a project for the city. A judge ordered the contractor to pay a fine of $2000 for day 1, $2500 for day 2, $3000 for day 3 and so on, until the project was completed. The contractor ultimately paid a find of $92,000. How late was the project finished?

5. Research conducted by an advertising agency suggests that, of every 50 people who learn about a new product, three will tell someone else about the product. Suppose a television commercial about a new brand of jeans reaches 3 million people. If the research is correct, how many people will learn about the jeans through the ad and word-of-mouth?

6. A ping-pong ball is dropped from a height of 16 ft. After the first bounce, the ball rebounds to a height of 4 ft. After the second bounce, the ball rebounds to a height of 1 ft. After the third bounce, the ball rebounds to a height of $\frac{1}{4}$ ft.

   a. Find the formula that represents the height of the ball on the nth bounce.

   b. Use the formula to find the height of the ball after the 6^{th} bounce.

7. Fred read about a way to become a millionaire. You save a penny the first day. Each day thereafter you save double the amount you saved the day before. Using this method , how much money would Fred save in a 20 day period?

Advanced Algebra - Sequences & Series
Date _____ REVIEW

Name _____

1. Find the 1st term of an arithmetic sequence if the 30th term is 122 and the difference is 4.

2. Find the 6th term in a geometric sequence if $a_1 = -3$ and $r = 2$.

3. Find three geometric means between 3 and 48.

4. If the sum of 20 terms in an arithmetic series is 1050 and the first term is 5, find the 20th term.

5. Write in expanded form.

a. $\sum_{K=3}^{5} (2k + 1)$

b. $\sum_{n=0}^{3} 3^n$

6. Write in summation notation: $2 + 5 + 8 + 11 + 14$

Advanced Algebra - Summation Notation Name _____
Assignment # ____ REVIEW

1. State the given series in expanded form.

a. $\sum_{n=1}^{4} 2n$

b. $\sum_{K=4}^{7} (k+2)$

c. $\sum_{n=0}^{3} 2^n$

d. $\sum_{n=1}^{4} 2(3^n)$

e. $\sum_{m=2}^{6} (3-2m)$

f. $\sum_{n=1}^{5} (-2)^n$

2. Identify which series in problem 1 are arithmetic or geometric.

a. arithmetic series _____

b. geometric series _____

3. Write each of the following using summation notation.

a. $6 + 8 + 10 + 12$

b. $8 + 1 - 6 - 13 - 20$

c. $3 + 9 + 27 + 81$

d. $1 + 5 + 25 + 125$

Arithmetic or geometric sequence and series review problems

4. Identify whether the example is an arithmetic sequence or series, a geometric sequence or series or none of them.

a. $\frac{1}{2}, 1, 2, 4$ _____

b. $11, 8, 5, 2$ _____

c. $8 + 4 + 0 + -4$ _____ d. $1, 0.1, 0.01, 0.001$ _____

e. $7, 7, 7, 7$ _____ e. $12 + 6 + 3 + 3/2$ _____

5. In a geometric sequence where $a_1 = 5$ and $r = -2$, find a_6.

6. In an arithmetic sequence where $a_5 = 1$ and $a_9 = 7$, find a_1.

7. Find two geometric means between 108 and -4.

8. Find the geometric series where $a_1 = 125$, $r = -2/5$, and $n = 5$.

9. Find the 1st term of a geometric series where $S_6 = 3640$ and $r = 3$.

10. Find the number of terms in an arithmetic series where $S_n = 33$, $a_1 = -12$, and $d = 3$.

WARMUP

Advanced Algebra - Sequences & Series Name Key

Date _____

1. The first two terms of a sequence are 7 and 14.

   a. Write the first 5 terms of the sequence if the sequence is arithmetic.

   7, 14, 21, 28, 35

   b. Write the first 5 terms of the sequence if the sequence is geometric.

   7, 14, 28, 56, 112

2. A small object that is dropped from the top of a tall building falls 16 ft during the first second, 48 ft during the second, 80 ft during the third second, 112 ft during the fourth second, and so on. $d = 32$

   a. Find the formula that will let you calculate the distance the object falls during the nth second.

   $a_n = 16 + (n-1)32$

   $a_n = 16 + 32n - 32$

   $a_n = 32n - 16$

   b. Use the formula to determine the distance the object falls during the 10^{th} second.

   $a_{10} = 32(10) - 16$

   $a_{10} = 320 - 16$

   $a_{10} = 304$ ft

3. The first row in a semicircular arena contains 72 seats. Moving back, each row has two more seats than the row in front of it. How many seats are there in the 50-row arena?

   $a_1 = 72 \quad d = 2 \quad n = 50$

   $S_{50} = \frac{50}{2}[2 \cdot 72 + (50-1)(2)]$

   $= 25[144 + 98]$

   $= 25(242)$

   $S_{50} = 6050$

4. A contractor failed to meet his deadline for completing a project for the city. A judge ordered the contractor to pay a fine of $2000 for day 1, $2500 for day 2, $3000 for day 3 and so on, until the project was completed. The contractor ultimately paid a find of $92,000. How late was the project finished?

$a_1 = 2000 \quad d = 500 \quad S_n = 92000$

$92000 = \frac{n}{2}[2 \cdot 2000 + (n-1)(500)]$

$92000 = \frac{n}{2}[4000 + 500n - 500]$

$92000 = \frac{n}{2}[500n + 3500]$

$92000 = 250n^2 + 1750n$

$0 = n^2 + 7n - 368$

$0 = (n+23)(n-16)$

$n = -23, 16$

$n = 16$

5. Research conducted by an advertising agency suggests that, of every 50 people who learn about a new product, three will tell someone else about the product. Suppose a television commercial about a new brand of jeans reaches 3 million people. If the research is correct, how many people will learn about the jeans through the ad and word-of-mouth?

$3,000,000 \div 50 = 60,000 \times 3 = 180,000$ word of mouth

$3,000,000$ ad

$a_1 = 3,000,000$

$r = \frac{3}{50}$

$a_2 = 180,000$

$S_2 = \frac{3,000,000(1-\frac{3}{50}^2)}{1-3/50}$

$S_2 = 3,180,000$

3,180,000

3,152,000

6. A ping-pong ball is dropped from a height of 16 ft. After the first bounce, the ball rebounds to a height of 4 ft. After the second bounce, the ball rebounds to a height of 1 ft. After the third bounce, the ball rebounds to a height of $\frac{1}{4}$ ft.

a. Find the formula that represents the height of the ball on the nth bounce.

$a_n = 4\left(\frac{1}{4}\right)^{n-1}$ a_1 = after 2nd bounce

$a_n = 16\left(\frac{1}{4}\right)^n$

b. Use the formula to find the height of the ball after the 6th bounce.

$a_6 = 4\left(\frac{1}{4}\right)^{6-1}$

$= .00390625$ ft

$a_6 = 16\left(\frac{1}{4}\right)^6$

$= .00390625$ ft

7. Fred read about a way to become a millionaire. You save a penny the first day. Each day thereafter you save double the amount you saved the day before. Using this method , how much money would Fred save in a 20 day period?

$.01 = a_1$

$r = 2$

$n = 20$

$S_{20} = \frac{.01(1-2^{20})}{1-2}$ = $10,485.75

Advanced Algebra - Sequences & Series
Date _____ REVIEW

Name Key

1. Find the 1st term of an arithmetic sequence if the 30th term is 122 and the difference is 4.

$a_1 = ?$ $a_{30} = 122$ $n = 30$ $d = 4$

$a_n = a_1 + d(n-1)$
$122 = a_1 + 4(30-1)$
$122 = a_1 + 4(29)$
$122 = a_1 + 116$
$6 = a_1$

2. Find the 6th term in a geometric sequence if $a_1 = -3$ and $r = 2$.

$a_6 = ?$ $n = 6$ $a_1 = -3$ $r = 2$

$a_n = a_1 \cdot r^{n-1}$
$a_6 = -3(2)^{6-1}$
$a_6 = -3(2)^5$
$a_6 = -3(32)$
$a_6 = -96$

3. Find three geometric means between 3 and 48.

find r

3, ___, ___, ___, 48
a_1 a_2 a_3 a_4 a_5
$n = 5$

$a_n = a_1 \cdot r^{n-1}$
$48 = 3 \cdot r^{5-1}$
$48 = 3 \cdot r^4$
$16 = r^4$
$\sqrt[4]{16} = \sqrt[4]{r^4}$
$2 = r$

6, 12, 24

4. If the sum of 20 terms in an arithmetic series is 1050 and the first term is 5, find the 20th term.

$a_{20} = ?$ $S_{20} = 1050$ $n = 20$ $a_1 = 5$

$S_n = \frac{n}{2}(a_1 + a_n)$
$1050 = \frac{20}{2}(5 + a_{20})$
$\frac{1050}{10} = \frac{10}{10}(5 + a_{20})$
$105 = 5 + a_{20}$
$100 = a_{20}$

5. Write in expanded form.

a. $\sum_{k=3}^{5} (2k+1) =$ 7+9+11

$2(3)+1$
$2(4)+1$
$2(5)+1$

b. $\sum_{n=0}^{3} 3^n =$ 1+3+9+27

3^0
3^1
3^2
3^3

6. Write in summation notation: $2 + 5 + 8 + 11 + 14$

$\sum_{n=1}^{5} 3n-1$

$a_1 = 2$
$d = 3$
$n = 5$

$a_n = a_1 + d(n-1)$
$a_n = 2 + 3(n-1)$
$a_n = 2 + 3n - 3$
$a_n = 3n - 1$

Advanced Algebra – Summation Notation Name Key
Assignment # ____ REVIEW

1. State the given series in expanded form.

a. $\sum_{n=1}^{4} 2n = 2+4+6+8$ ← $2(1), 2(2), 2(3), 2(4)$

b. $\sum_{k=4}^{7} (k+2) = 6+7+8+9$ ← $4+2, 5+2, 6+2, 7+2$

c. $\sum_{n=0}^{3} 2^n = 1+2+4+8$ ← $2^0, 2^1, 2^2, 2^3$

d. $\sum_{n=1}^{4} 2(3^n) = 6+18+54+162$ ← $2(3)^1, 2(3)^2, 2(3)^3, 2(3)^4$

e. $\sum_{m=2}^{6} (3-2m) = -1+-3+-5+-7+-9$ ← $3-2(2), 3-2(3), 3-2(4), 3-2(5), 3-2(6)$

f. $\sum_{n=1}^{5} (-2)^n = -2+4+-8+16+-32$ ← $(-2)^1, (-2)^2, (-2)^3, (-2)^4, (-2)^5$

2. Identify which series in problem 1 are arithmetic or geometric.

a. arithmetic series a, b, e

b. geometric series c, d, f

3. Write each of the following using summation notation.

a. a_1 6 + 8 + 10 + 12 $d=2$, $n=4$

$\sum_{n=1}^{4} 2n+4$

$a_n = a_1 + d(n-1)$
$a_n = 6+2(n-1)$
$a_n = 6+2n-2$
$a_n = 2n+4$

b. a_1 8 + 1 − 6 − 13 − 20 $d=-7$ $n=5$

$\sum_{n=1}^{5} -7n+15$

$a_n = a_1 + d(n-1)$
$a_n = 8+-7(n-1)$
$a_n = 8-7n+7$
$a_n = -7n+15$

c. a_1 3 + 9 + 27 + 81 $r=3$ $n=4$

$\sum_{n=1}^{4} 3(3)^{n-1}$

$a_n = a_1 \cdot r^{n-1}$
$a_n = 3(3)^{n-1}$
or 3^n

d. a_1 1 + 5 + 25 + 125 $r=5$ $n=4$

$\sum_{n=1}^{4} 1(5)^{n-1}$

$a_n = a_1 \cdot r^{n-1}$
$a_n = 1(5)^{n-1}$
or 5^{n-1}

Arithmetic or geometric sequence and series review problems

4. Identify whether the example is an arithmetic sequence or series, a geometric sequence or series or none of them.

a. $\frac{1}{2}, 1, 2, 4$ (×2 ×2 ×2) Geom. Sequence

b. 11, 8, 5, 2 (−3 −3 −3) Arithmetic sequence

c. $8 + 4 + 0 + -4$ (−4 −4 −4) Arith. Series

d. $1, 0.1, 0.01, 0.001$ ($\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$) Geom. Sequence

e. $7, 7, 7, 7$ (+0 +0 +0; ×1 ×1 ×1) Arith, Geom. Sequence

e. $12 + 6 + 3 + 3/2$ ($\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$) Geom. Series

5. In a geometric sequence where $a_1 = 5$ and $r = -2$, find a_6. $n = 6$

$a_n = a_1 \cdot r^{n-1}$

$a_6 = 5(-2)^{6-1}$

$a_6 = 5(-2)^5$

$a_6 = 5(-32)$

$a_6 = -160$

6. In an arithmetic sequence where $a_5 = 1$ and $a_9 = 7$, find a_1.

$a_n = a_1 + d(n-1)$

a_5	a_6	a_7	a_8	a_9
1	—	—	—	7
a_1				a_5

2nd: $7 = a_1 + 1.5(9-1)$

$7 = a_1 + 1.5(8)$

$7 = a_1 + 12$

$a_1 = -5$

1st: $7 = 1 + d(5-1)$

$7 = 1 + 4d$

$6 = 4d$

$d = 1.5$

7. Find two geometric means between 108 and −4.

$a_n = a_1 \cdot r^{n-1}$

$-4 = 108 \cdot r^{4-1}$

$-4 = 108 \cdot r^3$

$\sqrt[3]{\frac{-1}{27} = r^3}$

$-1/3 = r$

$108, \_\_, \_\_, -4$ (a_1, a_4)

$-36, 12$

8. Find the geometric series where $a_1 = 125$, $r = -2/5$, and $n = 5$.

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$S_5 = \frac{125(1-(-\frac{2}{5})^5)}{(1-(-2/5))}$

$S_5 = \frac{125(1-\,-.01024)}{1.4}$

$S_5 = \frac{125(1.01024)}{1.4}$

$S_5 = \frac{126.28}{1.4} = 90.2$

9. Find the 1st term (a_1) of a geometric series where $S_6 = 3640$ ($n = 6$) and $r = 3$.

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$3640 = \frac{a_1(1-(3)^6)}{(1-3)}$

$3640 = \frac{a_1(1-729)}{-2}$

$-7280 = a_1(-728)$

$10 = a_1$

10. Find the number of terms (n) in an arithmetic series where $S_n = 33$, $a_1 = -12$, and $d = 3$.

$S_n = \frac{n}{2}[2a_1 + d(n-1)]$

$33 = \frac{n}{2}[2(-12) + 3(n-1)]$

$33 = \frac{n}{2}[-24 + 3n - 3]$ ·2

$66 = n[3n - 27]$

$66 = 3n^2 - 27n$

$0 = 3n^2 - 27n - 66$

$0 = 3(n^2 - 9n - 22)$

$0 = 3(n-11)(n+2)$

$n = 11$ or $n = -2$

$n = 11$